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A survey of the introduction of rare-earth nuclei into the BETA-BEAM accelerator chain in order to attain a monochromatic neutrino beam.

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Abstract

With the recent discovery of rare-earth nuclei which decay fast through electron capture to single final states in Gamow Teller resonances, interest has ignited in the possibility of using the beta-beam accelerator facility to produce a monochromatic beam of electron neutrinos. This study is part of a Summer Studentship in the Beta-beam project in EURISOL DS and evaluates the ion intensities and incoherent Laslett tune-shifts in each machine for four rare-earth ions ^{148}Dy, ^{150}Dy, ^{150}Ho and ^{152}Tm. The simulation of the accelerator chain is a modified version of the Mathematica notebook intensities.nb [*see Appendix*], incorporating the four nuclei investigated, accumulation at low energy in the RCS and a vacuum stripping half-life of 60 seconds.

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1 Introduction to the Monochromatic Beta-Beam and Rare-Earth Nuclei

The monochromatic beta-beam idea is being investigated by Mats Lindroos (CERN) and Jose Bernabeu (Valencia University) along with their collaborators. The physics reach of such a neutrino beam is extremely promising for high precision experimental research into neutrino oscillation phenomena and CP violation in the lepton sector [1]. The monochromatic nature of such a beam originates from the two-body electron capture decay of a nucleus. An atomic electron is captured by a proton, resulting in the replacement of the nuclear proton with a neutron and the ejection of a neutrino from the system. The electron capture process is usually slow and able to branch to many final states, with many discrete values for neutrino energy, but recently some nuclei have been found to decay fast to a single final state [2]. The pure beam of monochromatic neutrinos produced, when boosted to a Lorenz factor γ in the accelerator chain, will have an energy, $E = 2\gamma E_0$, where E_0 is the energy of the neutrino in the nuclear rest frame [1]. The monochromatic neutrino beam energy is then tuneable by adjusting the energy of the ions in the decay ring, allowing for energy dependent measurements of the oscillation probability at a detector over a fixed baseline.

The nuclei used in this study were selected by J Bernabeu et al. for their suitability in such an accelerator complex and are shown in Table 1 [1]. The necessity of atomic electrons for the electron capture process to occur in the decay ring of the accelerator chain requires the acceleration and storage of partially charged ions. The acceleration of partially charge ions in the CERN accelerator complex has not been intensely studied and the vacuum conditions in PS and SPS would have to be upgraded. From work done so far it has been shown reasonable to use a vacuum half-life of 60 seconds.

Decay	$T_{1/2}$	BR_ν	EC/β^+	Q_{EC}	E_ν	ΔE_ν
^{148}Dy → ^{148}Tb*	3.1 m	1	96/4	2682	2062	0
^{150}Dy → ^{150}Tb*	7.2 m	0.64	100/0	1794	1397	0
^{152}Tm2$^-$ → ^{152}Er*	8.0 s	1	45/55	8700	4400	520
^{150}Ho2$^-$ → ^{150}Dy*	72 s	1	77/33	7400	3000	400

Table 1 – Four fast decays in the rare-earth region above ^{146}Gd leading to the giant Gamow-Teller resonance. Energies are given in keV. The first column gives the half-life, the second the branching ratio of decays to neutrinos, the third the relative branching between electron capture and β^+, the fourth is the neutrino energy and eighth its uncertainty. From reference [1].

2 Ion Intensities in the Accelerator Chain and Annual Rate Estimates

A source rate of 10^{13} ions per second from EURISOL is assumed and the intensities calculated for ions with a charge state of +50 are shown in Table 2. The dominant contribution to the electron capture process comes from the inner *s* electrons and the stripping of outer atomic electrons has very little effect on the process. The calculations are in line with the baseline scenario and include no accumulation. The energy in the decay ring corresponds to ions accelerated to a top Lorenz gamma of 100. For the electron capture decay of partially stripped ions in the decay ring, the annual rate R is given by,

$$R = \frac{I_{in} f}{T_{rep}} \times \frac{\lambda_{ec} / \gamma_{top}}{\lambda_{ec} / \gamma_{top} + \lambda_{vac}} \times (1 - e^{-mT_{rep}(\lambda_{ec}/\gamma_{top} + \lambda_{vac})}) \times T_{run},$$

where m is the number of merges that can be done in the decay ring without major losses from the merging process itself, T_{rep} is the repetition period for fills in the decay ring, γ_{top} is the gamma factor of the decay ring, λ_{ec} is the electron capture decay constant at rest, λ_{vac} is the equivalent vacuum decay constant, I_{in} the total number of ions injected into the decay ring for each fill, f the fraction of the decay ring length for the straight section generating the neutrino beam, and T_{run} the length in seconds of the run (extracted directly from [5]).

The annual rate estimates for the four rare-earth ions fall well short of the desired design rate of 10^{18} neutrinos per year per long straight section of the decay ring, although ^{152}Tm yields a rate equivalent to ^{18}Ne at baseline (with a source rate of 8 10^{11} s^{-1}) because of its short half-life.

	^{148}Dy	^{150}Dy	^{150}Ho	^{152}Tm
ECR [s^{-1}]	1.95 10^{11}	1.95 10^{11}	1.95 10^{11}	1.94 10^{11}
RCS Injection [s^{-1}]	9.74 10^{10}	9.74 10^{10}	9.74 10^{10}	9.70 10^{10}
PS Injection [s^{-1}]	1.92 10^{12}	1.92 10^{12}	1.92 10^{12}	1.83 10^{12}
SPS Injection [s^{-1}]	1.85 10^{12}	1.85 10^{12}	1.84 10^{12}	1.72 10^{12}
Decay Ring [s^{-1}]	1.93 10^{13}	1.94 10^{13}	1.92 10^{13}	1.61 10^{13}
Annual Neutrino Rate [yr^{-1}]	2.52 10^{15}	1.09 10^{15}	6.47 10^{15}	4.85 10^{16}

Table 2 – Intensity estimates for four rare-earth nuclei at the baseline scenario.

3 Space-Charge Effects of Rare-Earth Ions at Baseline

For a round Gaussian beam of ions, the self-field incoherent ("Laslett", see [3]) tune-shift ΔQ is defined as,

$$\Delta Q = -\frac{Z^2}{A_P} \frac{3r_p}{4\beta^3\gamma^3 c} \frac{R}{\tau_b} \frac{N_b}{\varepsilon},$$

where Z is the charge on the ion, A_p is the ion-to-proton mass ratio, r_p is the classical proton radius, c is the speed of light, R is the mean radius of the machine, N_b is the number of ions per bunch of duration τ_b and ε is the physical emittance of the beam. We assume τ_b is 80% of the bucket duration at injection in each machine. In all calculations involving the tune-shift, the values of emittance have been assumed to be of the same order as the emittances of ^{6}He and ^{18}Ne at baseline. The corresponding vertical tune-shifts calculated at baseline for the rare-earth ions are shown in Table 3 and are approximately three times as large as the Version 3 tune-shifts at baseline for ^{6}He and ^{18}Ne [4].

	^{148}Dy	^{150}Dy	^{150}Ho	^{152}Tm
RCS Injection	-0.155	-0.153	-0.153	-0.159
PS Injection	-0.482	-0.493	-0.493	-0.515
SPS Injection	-0.580	-0.595	-0.594	-0.614

Table 3 – Vertical tune-shifts of the rare-earth ions at injection in the three synchrotrons of the beta-beam accelerator chain.

The tune-shift values are large and any attempt to improve the annual rate through an increase in source rate or accumulation along the accelerator chain will only act to exacerbate the space-charge problem. The Z^2 dependence of the tune-shift is significant with such large ionic charges associated with the rare-earth ions. One way to reduce the machine tune-shifts would be to lower the charge state of the accelerated ions, hampering the top energy of the ions in the decay ring but increasing the annual neutrino rate. For ionic charges below +34 it is not possible to accelerate the rare-earth ions to an energy corresponding to a Lorenz gamma of 100 in the decay ring at the present energy of the SPS. The exact dependence of the vacuum stripping half-life on the ionic charge state is not known and in the following calculations it is assumed constant at 60 seconds as before. At lower charge states the outer electrons will be less tightly bound to the ion and more susceptible to ionisation. A balance between the limitations of space-charge and vacuum stripping needs to be found.

At a lower ionic charge of +25 and a top Lorenz gamma factor of 74 at SPS ejection, the tune-shifts and annual rate estimates are calculated as shown in Table 4.

	^{148}Dy	^{150}Dy	^{150}Ho	^{152}Tm
RCS Injection	-0.039	-0.039	-0.039	-0.039
PS Injection	-0.121	-0.123	-0.123	-0.125
SPS Injection	-0.145	-0.149	-0.148	-0.148
Annual Neutrino Rate [yr^{-1}]	2.7 10^{15}	1.2 10^{15}	7.1 10^{15}	5.6 10^{16}

Table 4 – Re-calculated tune-shifts for an ionic charge state of +25, γ_{TOP}=74.

The space-charge effect is more manageable at lower ionic charge but the effect on the vacuum stripping half-life is unknown and assumed constant. Even for the fastest decaying nucleus, ^{152}Tm, the integrated annual rate along one long straight section of the decay ring is 20 times below the design rate.

4 Accumulation in the Accelerator Chain

The beam intensities were optimised for the four rare-earth ions under investigation with accumulation along the accelerator chain, including low energy accumulation in the RCS. The ionic charge state was held at +25 and the top Lorenz gamma factor fixed at 100, which relies on an SPS ejection energy upgrade.

Rare-Earth Ion	ECR Shots	RCS Shots	PS Shots	Optimised Annual Rate [yr^{-1}]
^{148}Dy	23	1	14	6.76 10^{15}
^{150}Dy	20	9	1	3.88 10^{15}
^{150}Ho	11	19	1	2.01 10^{16}
^{152}Tm	6	20	1	1.16 10^{17}

Table 5 – Optimisation of annual rate for the rare-earth ions with accumulation.

ECR shots are merged together in the RCS to form one large bunch. The number of bunches in the PS refers to the number of RCS shots fired into it and the number of bunches in the SPS is given by the product of the RCS and PS shot number. In fact ^{152}Tm is not fully optimised but a slight sacrifice in annual rate was made for a different accumulation mode with much reduced space-charge effects. Even with accumulation in the facility the design rate of 10^{18} neutrinos per year is unrealistic with the four nuclei investigated. The most promising nucleus, ^{152}Tm, has the fastest half-life of 8.0 seconds but still produces a beam intensity one order of magnitude short of the design rate. A rare-earth nucleus with a half-life of order one second is required.

The intensities and respective tune-shifts along the accelerator complex for ^{152}Tm with accumulation are shown below; indicating that when accumulating along the accelerator chain, even at low charge states, space-charge is still unforgiving.

	Ion Intensity	Tune Shift
Source Rate [s^{-1}]	10^{13}	-
ECR	$1.94\ 10^{11}$	-
RCS Injection	$5.69\ 10^{11}$	-0.15
PS Injection	$7.45\ 10^{12}$	-0.50
SPS Injection	$6.65\ 10^{12}$	-0.58
Decay Ring	$3.36\ 10^{13}$	-
Annual Neutrino Rate [yr^{-1}]	$1.16\ 10^{17}$	-

Table 6 – Intensity values and tune-shifts for ^{152}Tm

5 Conclusion

The tune-shifts measured at the design rate for the acceleration of ^{6}He and ^{18}Ne in the beta-beam facility are manageable but huge problems arise when introducing the four rare-earth ions investigated here into the facility [4]. Unless a faster decaying ion is discovered then the design rate, even neglecting any space-charge limitations, will never be achieved. However, the annual rates achieved for the monochromatic beta-beams in this study could be useful in a near-detector experiment for calibration, promising a useful application for the monochromatic beam. A study into the acceleration of partially stripped ions in the beta-beam facility is required to understand the effects of reducing ionic charge on the vacuum stripping of the ions. In this way possible strategies for tackling the limitation of space-charge can be formulated. Further research into possible rare-earth candidates could yield an ideal candidate of shorter half-life.

6 Note

J. Bernabeu was concerned that the ^{152}Tm2^{-} nucleus decays by electron capture to two final states, which would disrupt the monochromatic nature of any beta-beam it produced. Two other isotopes of particular interest are ^{152m}Tm and ^{156}Yb but there was confusion in understanding the branching ratios in the data tables and insufficient time to complete an analysis of these nuclei [6].

7 References

[1] J. Bernabeu et al., *Monochromatic neutrino beams*, JHEP (2005) 014.
[2] E. Nacher, *Beta decay studies in the N~Z and the rare-earth regions using Total Absorption Spectroscopy techniques,* Ph. D. Thesis, University of Valencia, 2004.
[3] Parameter and Intensity Values, Version 2, July 2005 (doc-version), EURISOL DS/TASK12/TN-05-03.
[4] M. Fraser, *A Summer Studentship in the Beta-beam Task in Eurisol DS,* CERN.
[5] Mats Lindroos et al., *A Monochromatic neutrino beam,* Proc. Of Science.
[6] WWW Table of Radioactive Isotopes,
^{156}Yb, http://ie.lbl.gov/toi/nuclide.asp?iZA=700156
^{152m}Tm, http://ie.lbl.gov/toi/nuclide.asp?iZA=690452

Appendix – Modified Mathematica Notebook for Rare-Earth Nuclei

```
Off[General::spell1];
 c = 299792458;

(* All energies are in eV; "Tpern" = kinetic energy per nucleon. *)

setmachines := (
    bp = 1.2; (* 1.2s basic period. *)
    quantize[t_] := bp Ceiling[t/bp];
    ecrejTpern := 50 10^3 q/nnumber;
    ecraccumulationtime := rcscycletime - ecrdeadtime;
    ecrdeadtime = 2.5 10^-3;
    rcsinjTpern = 100 10^6;
    rcsefficiency = 0.5; (* Multi-turn injection. *)
    bunchingtime = 5 10^-3;
    rcsaccelerationtime := (rcscycletime - bunchingtime)/2;
    rcsBrhomax = 14.47; (* 3.5GeV proton equivalent. *)
    rcscycletime = 1/10;
    rcsbatches = 20;
    psinjTpern := Sqrt[(rcsBrhomax c q/nnumber)^2 + Epern^2] - Epern;
    psaccumulationtime := (rcsbatches-1) rcscycletime;
    psaccelerationtime = 0.8;
    psBrhomax = 86.7;
    pscycletime := quantize[psaccumulationtime + 2.0 psaccelerationtime + 0.1];
    psbatches = 1;
    spsinjTpern := Sqrt[(psBrhomax c q/nnumber)^2 + Epern^2] - Epern;
    spsaccumulationtime := (psbatches-1) pscycletime;
    spsaccelerationtime := (topTpern - spsinjTpern) * nnumber/(q 100 10^9);
       (* 100GeV/s proton equivalent. *)
    topgamma = 100;
    topTpern := Epern (topgamma - 1);
    spscycletime := quantize[spsaccumulationtime + 1.5 spsaccelerationtime + 1.0];
    spsrepetitiontime := Max[spscycletime, psbatches pscycletime];
    straightfraction = 0.36;
)

set148Dy:=(
    name="148Dy"; nnumber=148; q=25;(*Stripped of 50 electrons*)
    Erest=1.37793 10^11;(*Rest mass in eV/c^2.*)
    Epern:=Erest/nnumber;
    lambdaec=Log[2]/(3.1 60);(*Electron capture lambda*)
    lambdavac=Log[2]/60;
    tvac=60;(*Average vacuum half-life*)
    thalf=Log[2]/lambdaec;
    sourcerate=10^13;(*Effective rate,includes 40% efficiency.*)
    ecrefficiency=0.2;(*Single charge state.*)
    setmachines;
    setmods;
    mergesratio=15;
 )

 set150Dy:=(
    name="150Dy"; nnumber=150; q=25;
    Erest=1.39655 10^11; (*Rest mass in eV/c^2.*)
    Epern:=Erest/nnumber;
    lambdaec=Log[2]/(7.17 60);(*Electron capture lambda*)
    lambdavac=Log[2]/60;
    tvac=60; (*Average vacuum half-life*)
    thalf=Log[2]/lambdaec;
    sourcerate=10^13;(*Effective rate,includes 40% efficiency.*)
    ecrefficiency=0.2;(*Single charge state.*)
    setmachines;
    setmods;
    mergesratio=15;
 )
```

```
set150Ho:=(
    name="150Ho"; nnumber=150; q=25;
    Erest=1.39662 10^11; (*Rest mass in eV/c^2.*)
    Epern:=Erest/nnumber;
    lambdaec=Log[2]/(72);(*Electron capture lambda*)
    lambdavac=Log[2]/60;
    tvac=60; (*Average vacuum half-life*)
    thalf=Log[2]/lambdaec;
    sourcerate=10^13;(*Effective rate,includes 40% efficiency.*)
    ecrefficiency=0.2;(*Single charge state.*)
    setmachines;
    setmods;
    mergesratio=15;
 )

 set152Tm:=(
    name="152Tm"; nnumber=152; q=25;
    Erest=1.41535 10^11; (*Rest mass in eV/c^2.*)
    Epern:=Erest/nnumber;
    lambdaec=Log[2]/8.0 ;(*Electron capture lambda*)
    lambdavac=Log[2]/60;
    tvac=60; (*Average vacuum half-life*)
    thalf=Log[2]/lambdaec;
    sourcerate=10^13;(*Effective rate,includes 40% efficiency.*)
    ecrefficiency=0.2;(*Single charge state.*)
    setmachines;
    setmods;
    mergesratio=15;
)

setmods:=(
    ecrcycletime=1/10;
    ecraccumulationtime:=ecrcycletime-ecrdeadtime;
    ecrbatches=1;
    rcsaccumulationtime:=(ecrbatches-1) ecrcycletime;
    rcsaccelerationtime=0.0475;
    rcscycletime:=rcsaccumulationtime+bunchingtime+2.0 rcsaccelerationtime;
    rcsrepetitiontime:=Max[rcscycletime,ecrbatches ecrcycletime];
    psaccumulationtime:=(rcsbatches-1) rcsrepetitiontime;
    psrepetitiontime:=Max[pscycletime,quantize[rcsbatches rcsrepetitiontime]];
    spsaccumulationtime:=(psbatches-1) psrepetitiontime;
    spsrepetitiontime:=Max[spscycletime,psbatches psrepetitiontime];)

ecraccumulation := (ClearAll[n];
    gamma[t_] := 1 + ecrejTpern / Epern;
    decayrate[t_] := Log[2] n[t] / (((1/(gamma[t]*thalf))+(1/tvac))^(-1));
    eqns = {n'[t] == sourcerate - decayrate[t], n[0]==0};
    n[t_] = ecrefficiency n[t] /. DSolve[eqns, n[t], t] //First;
    nout0 = n[ecraccumulationtime]
)

rcsaccumulation := (ClearAll[n];
    gamma[t_] := 1 + rcsinjTpern / Epern;
    decayrate[t_] := Log[2] n[t] / (((1/(gamma[t]*thalf))+(1/tvac))^(-1));
    eqns = {n'[t] == -decayrate[t], n[0]==nout0};
    nsinglebatch[t_] = rcsefficiency n[t] /. DSolve[eqns, n[t], t] //First;
    n[t_] = Sum[UnitStep[t-t0] nsinglebatch[t,t0],{t0,0,rcsaccumulationtime,ecrcycletime}];
    nout0bis = n[rcsaccumulationtime]
)

rcsbunching := (ClearAll[n];
    gamma[t_] := 1 + rcsinjTpern / Epern;
    decayrate[t_] := Log[2] n[t] / (((1/(gamma[t]*thalf))+(1/tvac))^(-1));
    eqns = {n'[t] == -decayrate[t], n[0]==nout0bis};
    n[t_] = n[t] /. DSolve[eqns, n[t], t] //First;
    nout1 = n[bunchingtime]
)
rcsacceleration := (ClearAll[n];
    rcsTpern[t_] := rcsinjTpern + (psinjTpern - rcsinjTpern) (1 - Cos[Pi t/rcsaccelerationtime])/2;
    gamma[t_] := 1 + rcsTpern[t] / Epern;
    decayrate[t_] := Log[2] n[t] / (((1/(gamma[t]*thalf))+(1/tvac))^(-1));
    eqns = {n'[t] == -decayrate[t], n[0]==nout1};
    n[t_] = n[t] /. DSolve[eqns, n[t], t] //First;
    nout2 = Simplify[ n[rcsaccelerationtime] ]
)
rcsout := (ecraccumulation; rcsaccumulation; rcsbunching; rcsacceleration)
psaccumulation := (ClearAll[n];
    gamma[t_] := 1 + psinjTpern / Epern;
    decayrate[t_] := Log[2] n[t] / (((1/(gamma[t]*thalf))+(1/tvac))^(-1));
    eqns = {n'[t] == -decayrate[t], n[0]==nout2};
    nsinglebatch[t_] = n[t] /. DSolve[eqns, n[t], t] //First;
    n[t_] = Sum[UnitStep[t-t0] nsinglebatch[t-t0],{t0,0,psaccumulationtime,rcscycletime}];
    nout3 = n[psaccumulationtime]
)

psacceleration := (ClearAll[n];
    psTpern[t_] := psinjTpern + (spsinjTpern - psinjTpern) t/psaccelerationtime;
    gamma[t_] := 1 + psTpern[t] / Epern;
```

```
    decayrate[t_] := Log[2] n[t] / (((1/(gamma[t]*thalf))+(1/tvac))^(-1));
    eqns = {n'[t] == -decayrate[t], n[0]==nout3};
    n[t_] = n[t] /. DSolve[eqns, n[t], t] //First;
    nout4 = n[psaccelerationtime]
)
psout := (rcsout; psaccumulation; psacceleration)

spsaccumulation := (ClearAll[n];
    gamma[t_] := 1 + spsinjTpern / Epern;
    decayrate[t_] := Log[2] n[t] / (((1/(gamma[t]*thalf))+(1/tvac))^(-1));
    eqns = {n'[t] == -decayrate[t], n[0]==nout4};
    nsinglebatch[t_] = n[t] /. DSolve[eqns, n[t], t] //First;
    n[t_] = Sum[UnitStep[t-t0] nsinglebatch[t-t0],{t0,0,spsaccumulationtime,psrepetitiontime}];
    nout5 = n[spsaccumulationtime]
)


spsacceleration := (ClearAll[n];
    spsTpern[t_] := spsinjTpern +(topTpern - spsinjTpern) t/spsaccelerationtime;
    gamma[t_] := 1 + spsTpern[t] / Epern;
    decayrate[t_] := Log[2] n[t] / (((1/(gamma[t]*thalf))+(1/tvac))^(-1));
    eqns = {n'[t] == -decayrate[t], n[0]==nout5};
    n[t_] = n[t] /. DSolve[eqns, n[t], t] //First;
    nout6 = n[spsaccelerationtime]
)
spsout := (psout; spsaccumulation; spsacceleration)

decayringaccumulation := (ClearAll[n];
   gamma[t_] := 1 + topTpern / Epern;
   decayrate[t_] := Log[2] n[t] / (((1/(gamma[t]*thalf))+(1/tvac))^(-1));
   eqns = {n'[t] == -decayrate[t], n[0]==nout6};
   nsinglebatch[t_] = n[t] /. DSolve[eqns, n[t], t] //First;
   n[t_] = Sum[UnitStep[t-t0] UnitStep[t0+mergesratio*spsrepetitiontime-t] *
nsinglebatch[t-t0], {t0, 0,t,spsrepetitiontime}];
   nout7 = n[(mergesratio-1)spsrepetitiontime]
)
fullchain := (spsout; decayringaccumulation)

annualrate := (1 - 2^-(mergesratio spsrepetitiontime/(topgamma thalf))) * spsout
straightfraction 10^7/spsrepetitiontime

annualrateEC:=(1-Exp[-mergesratio spsrepetitiontime(lambdaec/topgamma+lambdavac)])* spsout
straightfraction lambdaec/topgamma 10^7/(spsrepetitiontime(lambdaec/topgamma+lambdavac));
```