AVF Accelerators – past, present and future

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Introduction The AVF class of fixed-field accelerators contains two families – the AVF cyclotrons and the FFAGs

Advantages of fixed magnetic field:

- Higher duty factor – up to 100%
- Higher time-averaged beam current
- Simpler and inexpensive power supplies
- No eddy-current effects, no cyclical stressing of coils

Principle disadvantage: particle beam moves radially – large aperture magnet, vacuum and radio-frequency systems

Over-arching trend towards designs with increasing momentum compaction – narrower apertures, but longer orbits.

$\alpha=(dp/p)/(dL/L)$ – Livingood definition of compaction

$\alpha \approx 1$ for Lawrence cyclotron; $\alpha \rightarrow \infty$ for Johnstone FFAG

Simple cyclotron

Homogeneous magnetic field B_z, circular orbits, approximate isochronism for non-relativistic particles, no vertical stability.

Gradient (or weak) focusing cyclotron

Radial variation of axial field
$B_z=B_0(r_0/r)^n$ n = field index

Vertical focusing and diminution of radial stability, respective betatron tunes satisfy

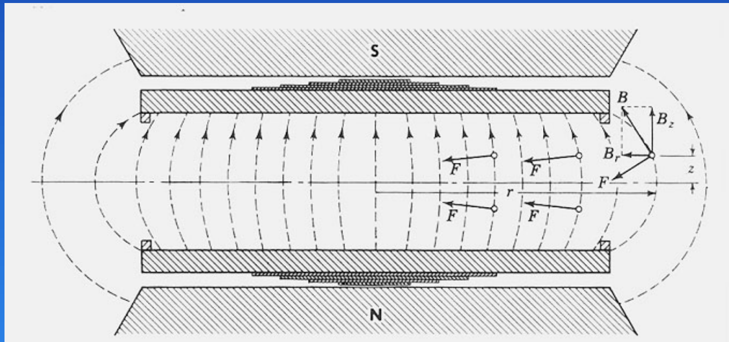


Fig. 6-7. Radially decreasing magnetic field between poles of a cyclotron magnet, showing shims for field correction.

$$\nu_z^2 = n \qquad \nu_r^2 = (1-n) \qquad 0 < n < 1$$

Positive index, radially falling field, opposite to requirement for isochronism. Frequency Modulation (FM) operation, $\alpha = 1-n$

Azimuthally Varying Field (AVF)

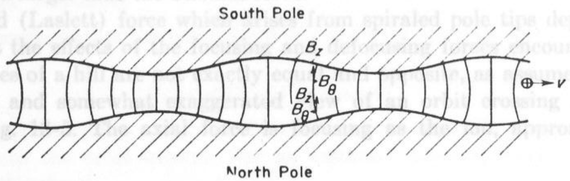


Fig. 13-1. Side view of sinusoidal pole tips and of field lines in a Thomas cyclotron which has been "unrolled" for clarity of illustration. The observer is at the center looking outward.

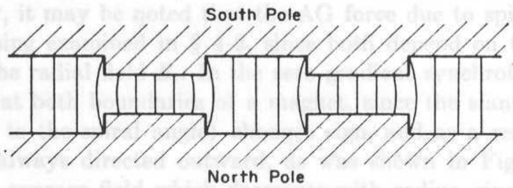


Fig. 13-2. Side view of an "unrolled" Thomas cyclotron with square-wave pole tips.

Leads to vertical stability even if there is no radial variation

Radial sector type

$$B_z = \langle B_z \rangle \left[1 - f \sin N\theta \right]$$

$$F = f^2/2$$

- Field bulges $\Rightarrow B_\theta$
- Scalloped orbit $\Rightarrow v_r$
- Lorentz force $F_z = q v_r B_\theta \Rightarrow$ *Thomas focusing*
- Vertical tune increased
- Radial tune unaffected

$$\nu_z^2 = f^2/2$$

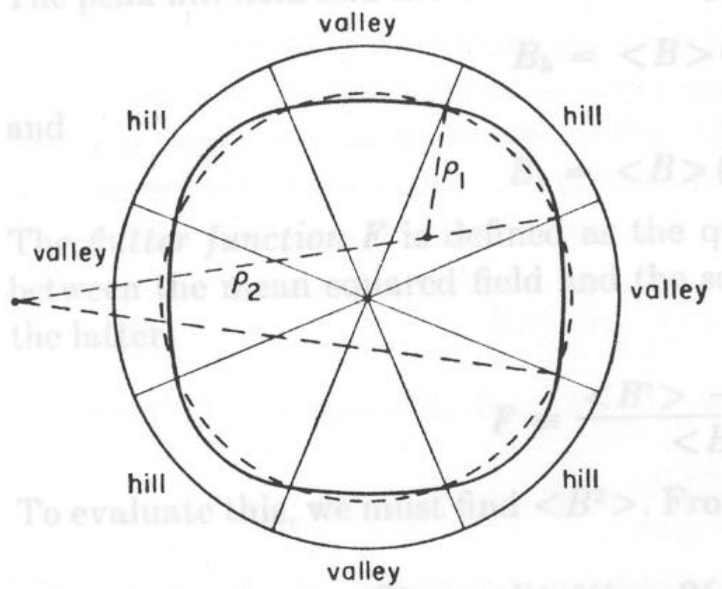


Fig. 13-3. Four-sector Thomas cyclotron with idealized square-wave variation of field. The orbit has two radii of curvature and forms scallops about a circle.

Azimuthally Varying Field – cont'd

Spiral Sector type

Loci of hill and valley extrema follow a spiral path

Thomas focusing is supplemented by Kerst-Laslett focusing, that is alternating-gradient edge focusing.

Magnet pole spiral angle $\xi(r)$

Radial field $\Rightarrow B_r = - B_\theta \tan\xi$

Angled orbit $\Rightarrow v_\theta$

Lorentz force $F_z = q v_\theta B_r \Rightarrow$ *Kerst-Laslett* focusing

Radial tune unaffected

Vertical tune increased $\nu_z^2 = F(1+2\tan^2\xi)$

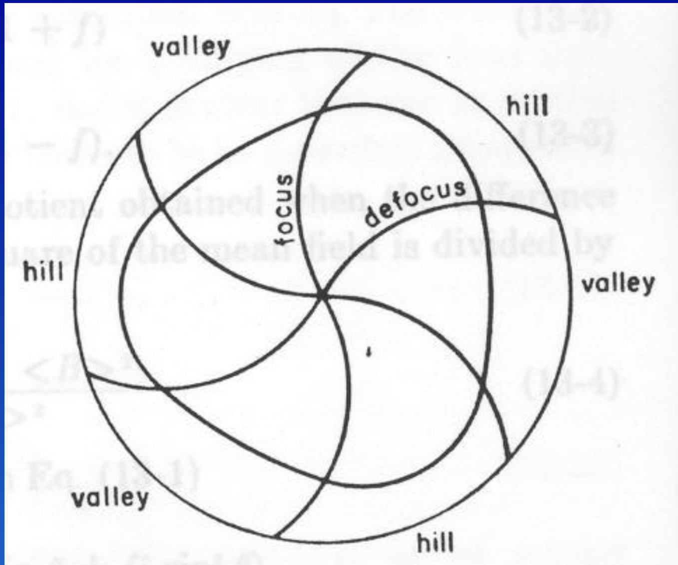
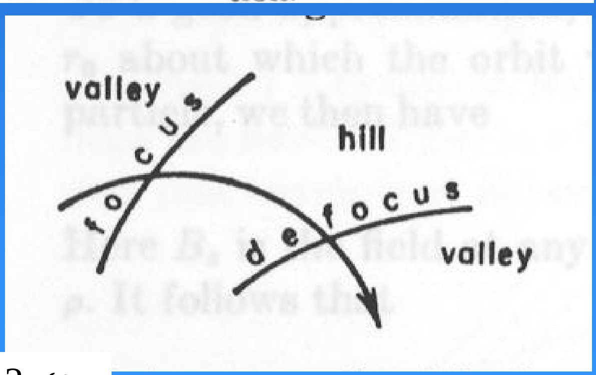


Fig. 13-4. When the spiral angle is large enough, axial focusing and defocusing forces occur at the boundaries between high and low fields, giving rise to Kerst alternating-gradient stabilization.



Gradient plus AVF focusing

When gradient is superposed on AVF, the field index has to be generalized because orbits are no longer circular

The betatron tunes become

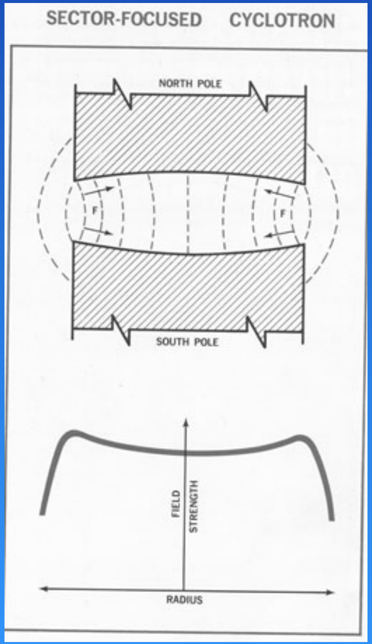
n= -(dB/B)/(dr/r); k=+<dB/B>/(dR/R)

$$\nu_z^2 = -k + \frac{N}{N^2 - 1} F(1 + 2\tan^2 \xi) + \ldots$$

$$\nu_r^2 = 1 + k + \ldots$$

$$\alpha = k + 1$$

AVF cyclotron



Mixed weak and AVF focusing removes constraints on field index to yield an isochronous cyclotron. The magnetic field increases with radius to ensure isochronism, and the vertical (weak) defocusing is overcome by Thomas (& Kerst-Laslett) focusing. $k(r) = \gamma^2 - 1$ implies crossing of betatron resonances. Both radial and spiral sector machines built; and flutter f<1, so there are no sign changes of B_z.

Advantages of isochronism c.w. operation, simpler less expensive rf system, on-crest acceleration, pure resitive beam load of accelerating cavity.

AVF focusing plus gradient focusing continued - **FFAG**

Fixed-field, alternating-gradient (FFAG) accelerator differs from AVF cyclotron in two respects.

- Machines with extreme Thomas focusing – flutter so large (f >1) that magnetic field reverses sign between hills and valleys. This reversal never occurs in cyclotrons.

Corollary: the gradient part is made to alternate in sign from sector to sector, giving AG focusing whose residual is *strong* if the gradients are large.

$$B_z = \langle B_z(r)\rangle \left[1 - f \sin N\theta\right]$$

- Machines emphasise achromatic rather than isochronous performance – but this is not essential.

FM operation is usual.

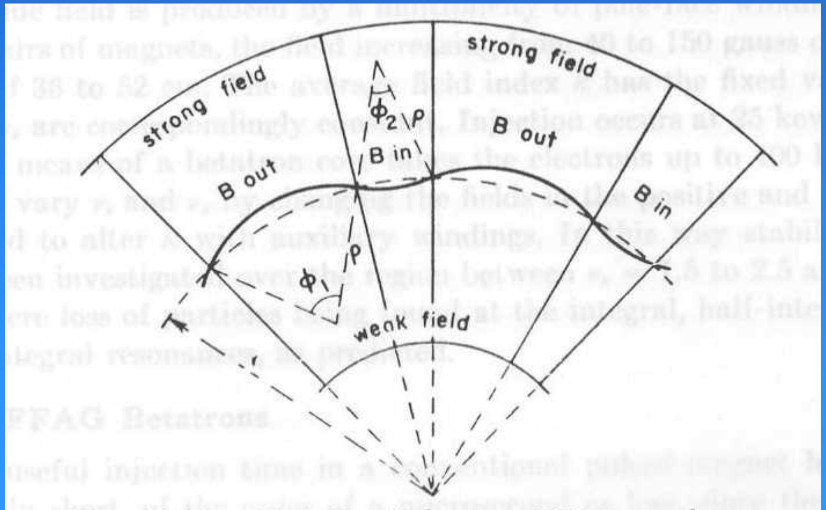


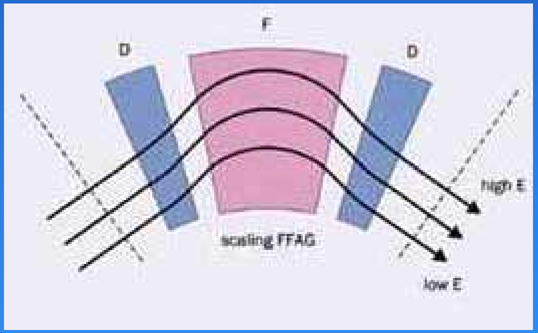
Fig. 13-16. Mark I radial sector FFAG accelerator.

Classical FFAG –continued

Both radial and spiral sector FFAGs have been proposed and constructed (MURA, etc). Weaker flutter in the spiral type reduces the machine circumference, and the reduced focusing (cf radial type) is restored by Kerst-Laslett forces due to the spiraling.

Momentum compaction $\alpha = (dp/p)/(dL/L) = k+1$ is relatively large in FFAGs, suggesting a ring-like layout – but this is not essential.

Scaling FFAGs



This special combination of sectoral layout and constant field index results in geometrically similar orbits – leading to zero chromaticity.

Advantage: no crossing of betatron resonances

Disadvantage: not isochronous – FM operation essential

Large physical apertures and zero chromaticity $\Rightarrow$ large 6D acceptance. But compromised by nonlinear resonances driven by inherent field multipoles – because of radial index.

Non-scaling FFAGs

The scaling FFAG occupies a narrow niche in design space.
In principle, every other design is non-scaling.

Modern FFAGs

The recent Japanese FFAGs are classical scaling designs infused with modern technical innovations. Instrumental in revitalising interest in FFAGs.

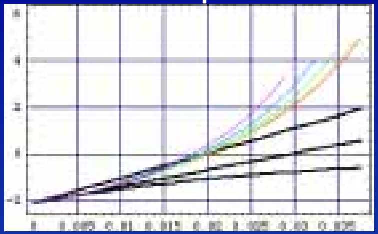
Truly modern designs have lattice layout more reminiscent of AG synchrotrons than AVF cyclotrons or scaling FFAGs.

Linear-field FFAG (C. Johnstone, FNAL)

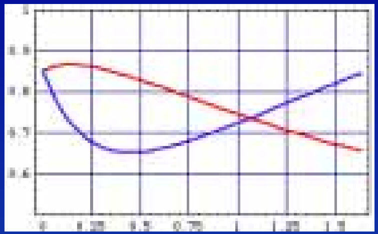
The linear-field, varying-tune, almost-isochronous FFAG of C. Johnstone has radial-sector FFAG and AG synchrotron antecedents. Lattice composed of alternating-polarity quadrupole magnets with drift spaces for cavities. Truly an FFAG because the bend fields and gradients reverse sign across a half cell. Optics devised to give infinite momentum compaction at mid energy. Operation is near isochronous.

Modern FFAGs -continued

F field profile

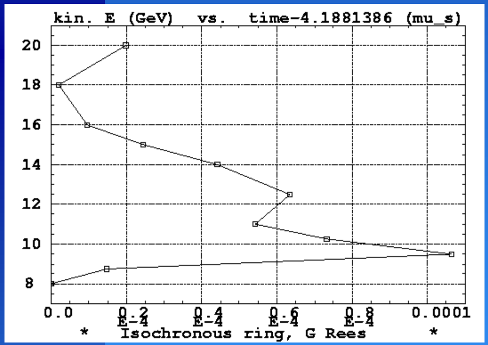


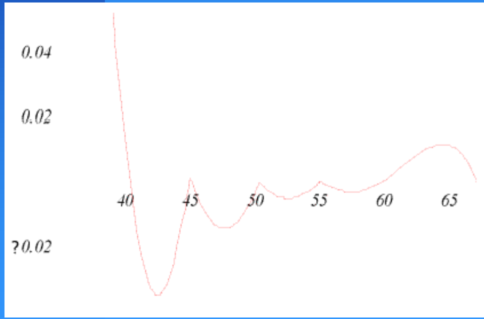
Betatron tunes



Adjusted field profile FFAG (A. Ruggiero)

In a synchrotron-like layout, this design uses nonlinear quadrupoles (field gradient varies in radius and azimuth) to give small negative chromaticity and large 6D acceptance. Non-isochronous, parabolic ToF, FM operation assumed; early version used microtron principle of jumping harmonic numbers. Acceptance may be compromised by nonlinear resonances.





ToF (ps) vs momentum (Horst)

FFAGs of Rees and Schoenauer

The lattices have several nonlinear elements and include deliberate geometrical features to promote edge focusing - they bear affinity to spiral-sector classical FFAGs. All parameters vary with radius.The Rees lattice has weak positive chromaticity and perfect isochronism. The Schoenauer lattice has almost zero chromaticity and near perfect isochronism. Both are true FFAGs – the bend fields and gradients alternate in sign across half cells.

Conclusions

The AVF class of fixed-field, circular accelerators, includes the AVF cyclotron family and the FFAG family. They express different degrees of a continuous variable, flutter, but operate in such different regimes that they are arguably distinct.

Both families have combined gradient and AVF focusing. The cyclotron ($f<1$) has only the weak-type of gradient focusing, whereas the FFAG ($f>1$) has the strong type of focusing because the gradients are made to alternate. The alternating bending fields make for larger circumference, but higher momentum compaction makes for smaller apertures.

The so-called “scaling” rings occupy a tiny fraction of the FFAG territory, and continued fruitful exploration of other designs can be expected for as long as motivation and resources are provided.

Several niche applications have been identified, the linear-field FFAG for muon acceleration, and the adjusted-field-profile FFAG for a proton driver.