^{6}He FFAG

Refs used here :

− calculations are based on Yoshimoto's methods, and Machida's paper "BEAM OPTICS DESIGN OF AN FFAG SYNCHROTRON"

− Japan NuFact design report

− KURRI documents

Present procedure, below, derived from similar exercise on 150 MeV FFAG : /home/fmeot/mathematica/accelerators/ffag/150MeV/150MeV.nb.

An example of an FFAG cascade, KURRI ADS/Reactor

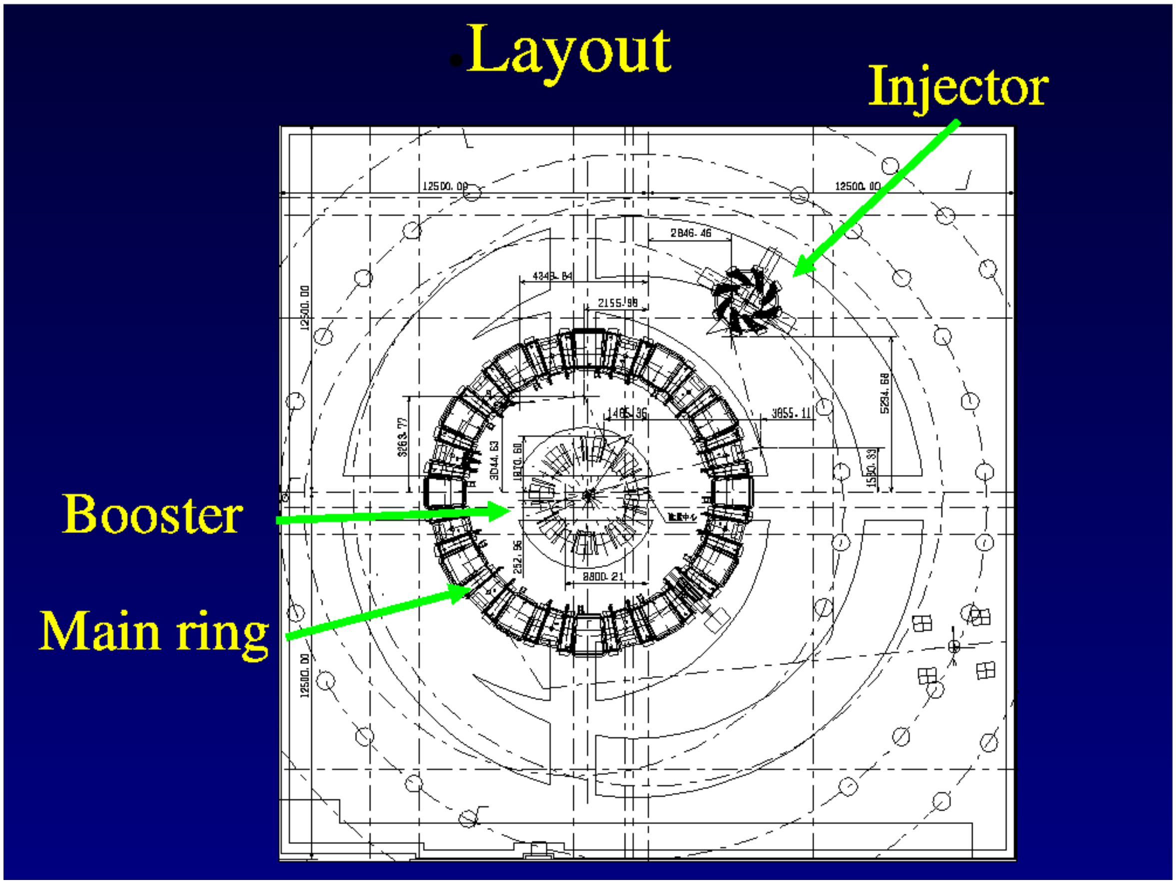
H^- 100 keV → 2.5 MeV → 20 MeV → 200 MeV

Advantage of FFAG :

. fast cycling

. large acceptance

```
data = Import["ishi1_Kurri-ADSR.gif"]
Show[Graphics[data]]
```



Possible momentum cascade for ${}^{6}_{2}He$, case of pole shaping

```
data = Import["popMagnet.gif"]
Show[Graphics[data]]
```

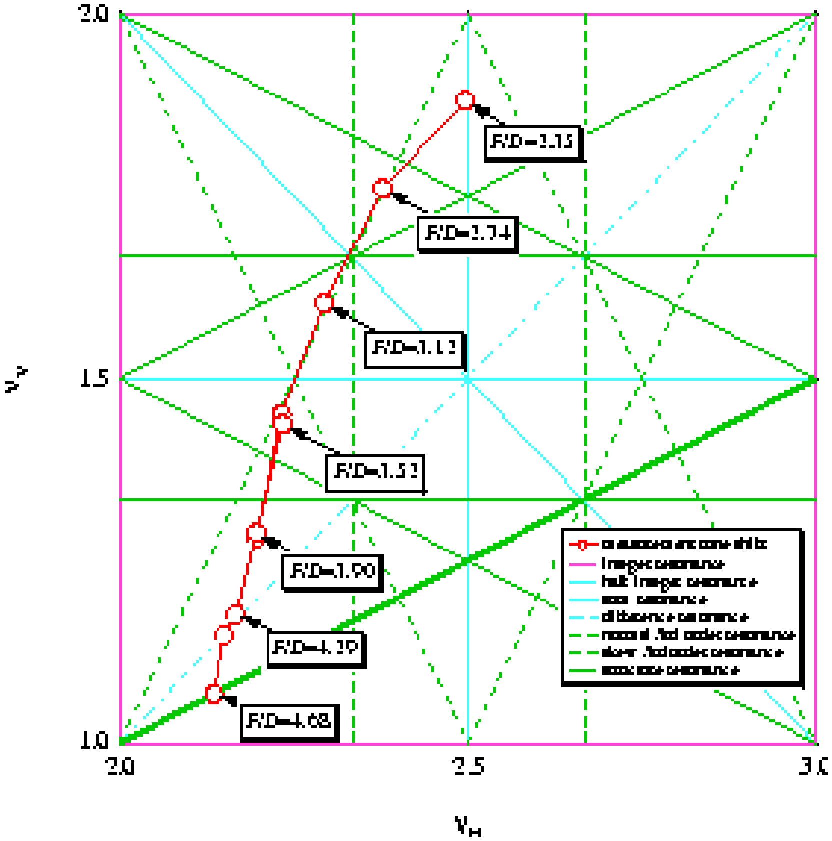
The gap is of the form $g = g_0 \, (R_0 \, / \, r)$ and yields $B = B_0 \, (r / R_0)^K$

```
data = Import["118-1825_IMG.gif"]
Show[Graphics[data]]
```

200 MeV ring :

```
data = Import["dNudB.gif"]
Show[Graphics[data]]
```

Tune control in the POP FFAG :




```
ClearAll["Global*`"]
c = 2.99792458 10^8 ;
mp =  938.27231 10^6 ;
amu =  931.49 10^6 ;
m = . ;
T = . ;
En = T + m ;
gamma = En / m ;
p = Sqrt[En^2 - m^2] ;
beta = p / En ;
betGam = beta gamma ;
A = .;
q = .;
BRho = p / (q / A) / c ;
```

```
T = {.1, 2.5, 20., 150. , 500.} 10^6 ;
m = amu ;
p / 10^6 ;
BRho /. {q → 2, A → 6} ;
% c / 10^6
% 10^6 ;
(* Ekin, proton equivalent :            *)
(Sqrt[% % + mp mp]  - mp ) / 10^6
Table[
 BRho[[i + 1]] / BRho[[i]], {i, 1, Length[T] - 1}
]
```

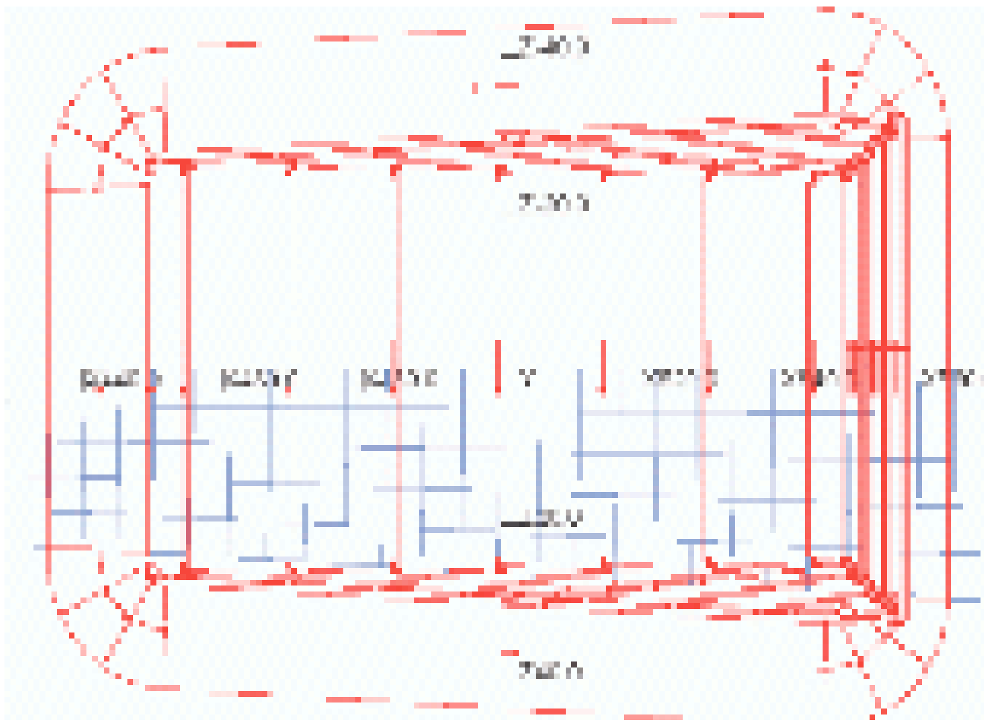
```
{40.9484, 204.874, 582.182, 1648.49, 3260.89}

{0.893117, 22.1069, 165.943, 958.535, 2454.92}

{5.00322, 2.84166, 2.83157, 1.97811}
```

Possible momentum cascade for ${}^{6}_{2}He$, case of coil shaping ?

The gap is flat,
the coil distribution yields $B = B_0 (r / R_0)^K$.
Allows control of both F / D ratio (ν_z) and K value (ν_x).

```
data = Import["coilShaping.gif"]
Show[Graphics[data]]
```

```
- Graphics -
```



```
- Graphics -
```

```
T = {.1, 2.5, 40., 500.} 10^6 ;
m = amu ;
p / 10^6 ;
BRho /. {q → 2, A → 6}
% c / 10^6;
% 10^6 ;
(* Ekin, proton equivalent :             *)
(Sqrt[% % + mp mp] - mp ) / 10^6
Table[
 BRho[[i + 1]] / BRho[[i]], {i, 1, Length[T] - 1}
]
```

```
{0.136589, 0.683386, 2.76088, 10.8772}

{0.893117, 22.1069, 312.898, 2454.92}

{5.00322, 4.04, 3.93974}
```

DESIGN OF A CELL

Working hypothesis

```
<< Graphics`MultipleListPlot`
ClearAll["Global`*"]
```

Hypothesis, particle

```
amu = 931.49 10^6 ;
c = 2.99792458 10^8 ;
m = .;   T = .;
En = T + m;   gamma = En / m;  p = Sqrt[En^2 - m^2];
beta = p / En;  betGam = beta gamma;
A = .; q = .; ruleBRho = p / (q / A) / c;
m = amu ; Tmin = 40. 10^6 ; Tmax = 500. 10^6 ;
T = Tmax ; BRhoMax = ruleBRho /. {q → 2., A → 6.}
T = Tmin ; BRhoMin = ruleBRho /. {q → 2., A → 6.}
```

```
10.8772
```

```
2.76088
```

Hypothesis, magnet field / geometry

The max field in F magnet is BF0 = 1.8 T.

```
BF0 = 1.8  (* max field at max R0 *)
magneticLength = BRho * 2 * Pi / BF0;
lBend = 3.5 ;
FDR = 1.366
(* F/D ratio =
  betF/betD with betF= half-sector angle of F magnet,
 betD =  sector angle of D magnet  *) ;
lF = . ; lD = . ;  (* half lengths in F and D poles *)
listLengths = {lF, lD} /. Solve[
   {2 (lF + lD ) == lBend, lF == FDR * lD}, {lF, lD}] ;
lengths = Flatten[listLengths] ;
lF = lengths[[1]] ; lD = lengths[[2]] ;
FDFieldRatio = 1.09 ;
lMagEffective =  2 ( lF - lD ) * FDFieldRatio
magneticLength / lMagEffective ;
nCell = Round[magneticLength / lMagEffective]
angSectorCell = 2. Pi / nCell  ;
packFac =  0.45  ;
R0 = nCell * lBend / (2. Pi)  / packFac
betF = . ; betD = . ; bet0 = . ;
listAngles = {betF, betD, bet0} /.
   Solve[{2 (betF + betD + bet0) == angSectorCell,
     2 R0 (betF + betD)  == lBend, betD == betF / FDR},
    {betF, betD, bet0}] ;
angles = Flatten[listAngles]
angles * 180 / Pi
betF = angles[[1]];
betD = angles[[2]]; bet0 = angles[[3]];
```

```
1.8

0.59296

64

79.6765

{0.0127516, 0.00933776, 0.0269981}

{0.730611, 0.535014, 1.54688}
```

Now, define K value :

```
a = 0.33 ;
orbitExcursion = 0.46 ;
Rmin = R0 - orbitExcursion;
BFmin = a * BF0  ;
U =. ;
listK = U /. Solve[BF0 / BFmin == (R0 / Rmin)^U, U] ;
K = listK[[1]]
Raverage = R0 - orbitExcursion / 2.
BFLength = 2.  * betF * Raverage
BDLength = betD * Raverage
driftLength = 2.  * Raverage * bet0
cellLength = 2. * (betF + betD + bet0 ) * Raverage
momentumCompac = 1. / (1. + K)
gammaTr = (1 / momentumCompac)^0.5
```

```
Solve::ifun : Inverse functions are being used by Solve, so some
   solutions may not be found; use Reduce for complete solution information. More…
```

```
191.476

79.4465

2.02613

0.741852

4.2898

7.79964

0.00519544

13.8736
```

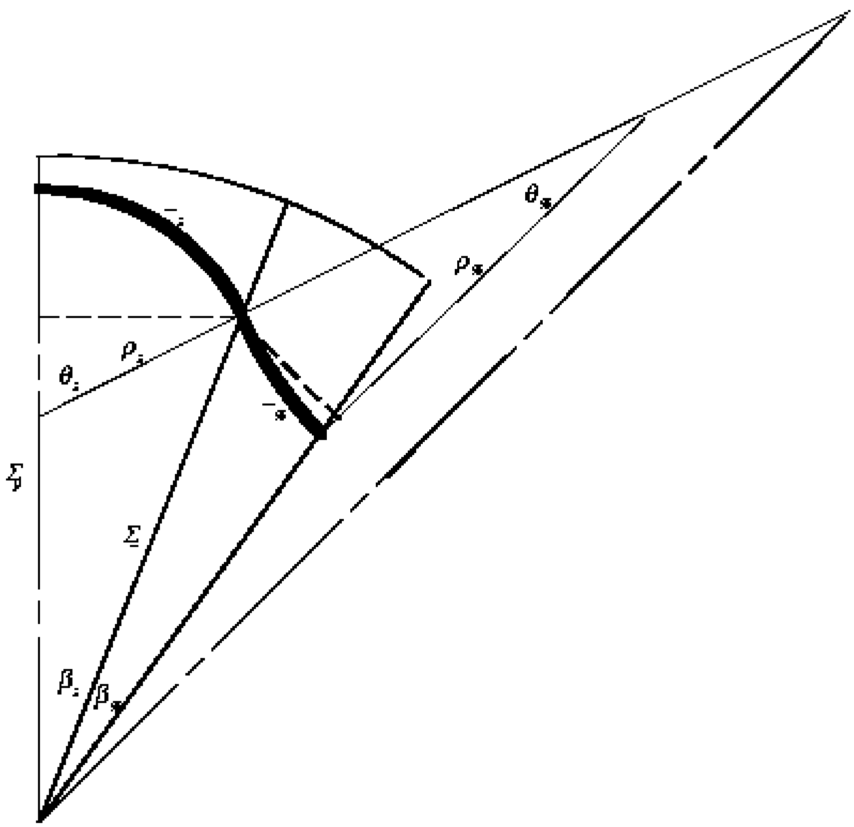
■ Hypothesis, ring geometry

One choses the F/D angle ratio, $\gamma=-\beta F/\beta D$, as large as possible, so to limit the circumference factor. on the other hand, $\gamma = \Gamma = {\psi_1}^2 / {\psi_2}^2$ with ψ_1, ψ_2 related to σ_1, σ_2 the H/V phase advance per cell. These should be around $\pi/2$. Choose for instance, $\sigma_1 = 2\pi/3$, $\sigma_2 = \pi/3$. ..

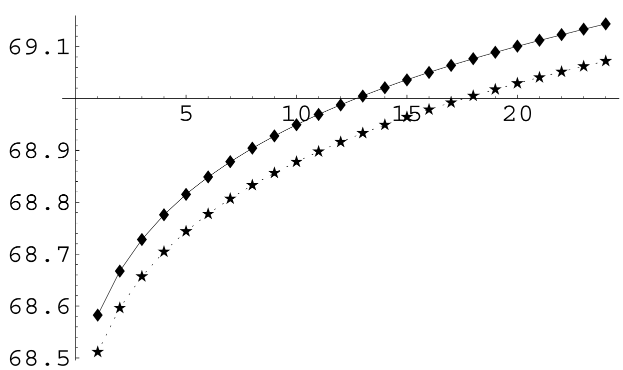
Calculation of ingredients for matrix formalism

- **compute**
  r_F (geometrical radius in middle of the F magnet),
  r_D (geometrical radius between F and D magnets) and
  θ_F (bend angle of half the F magnet). If needed, θ_D ensues (bend angle of the D magnet) : $\theta_D + \pi/\text{nCell} + (\pi - \theta_F) = \pi$.

```
data = Import["geometry.gif"]
Show[Graphics[data]]
```



```
Clear[E1, E11, E2, E21, E3, E31, T, BRho];
tetF =. ;
tetD = . ;
rF = . ;
rD = . ;
E1 := rhoF (Sin[tetF] + (1 - Cos[tetF] ) Tan[betF]) /
     Tan[betF] - rF  ;
E2 := rhoF  Sin[tetF] / Sin[betF] - rD  ;
E3 := -rhoD +
   rD (Sin[Pi / nCell - betF]  - Cos[Pi / nCell - betF]
        Tan[Pi / nCell - betF - betD])  / (Sin[
        tetF - Pi / nCell ]  - (1 - Cos[tetF - Pi / nCell] )
        Tan[Pi / nCell - betF - betD] ) ;
E11 := E1 /. rhoF → BRho / BF0  (R0 / rF)^K ;
E21 := E2 /. rhoF → BRho / BF0  (R0 / rF)^K ;
E31 := E3 /. rhoD → BRho / BD0  (R0 / rD)^K ;

listR = Table[
  (rF =.; tetF =.; rD =.;
   BRho := Sqrt[T (T + 2 * m)] / c ;
FindRoot[{E11 == 0, E21 == 0, E31 == 0}, {{rF, R0},
     {tetF, Pi / nCell + betF + bet0}, {rD, R0}}])
  , {T, 40. 10^6, 500. 10^6, 20 10^6}] ;
listRoot = {rF, rD} /. % ;
listrF = Table[
   Extract[listRoot, {i, 1}], {i, 1, Length[listR]}];
listrD = Table[Extract[listRoot, {i, 2}],
   {i, 1, Length[listR]}];
(* ListPlot[listrF, PlotJoined → True];
 ListPlot[listrD, PlotJoined → True];  *)
 MultipleListPlot[listrF, listrD, PlotJoined → True];
```

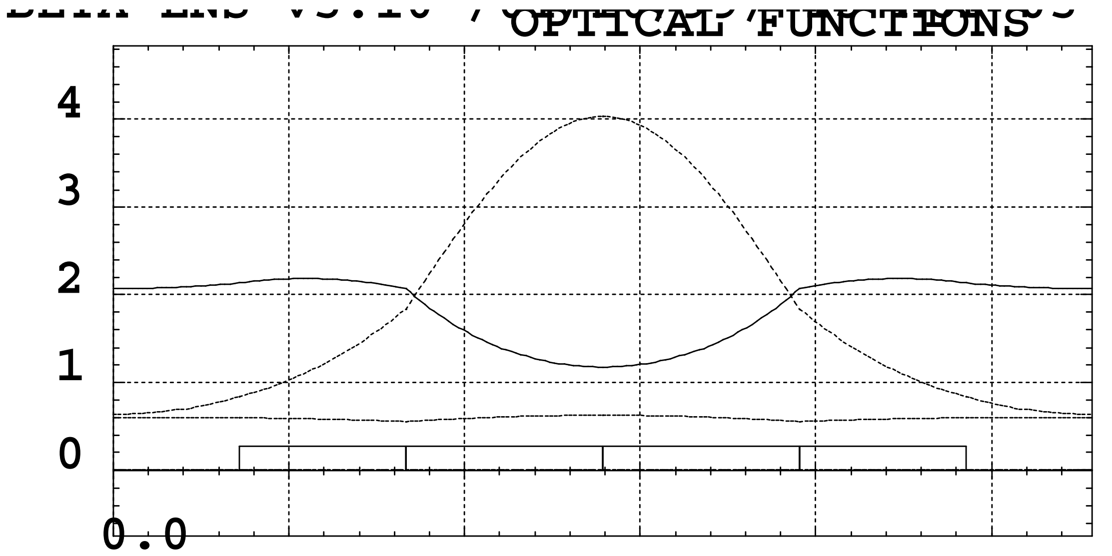


```
T = 500 10^6 ;
rF =.;
tetF =.;
rD =.;
BRho := Sqrt[T (T + 2 * m)] / c ;
FindRoot[{E11 == 0, E21 == 0, E31 == 0},
  {{rF, R0}, {tetF, Pi / nCell + betF + bet0}, {rD, R0}}];
listRoot = {rF, tetF, rD} /. % ;
rF = listRoot[[1]];
tetF = listRoot[[2]]
rD = listRoot[[3]];
rhoF = BRho / BF0 (R0 / rF)^K
rhoD = BRho / BD0 (R0 / rD)^K
tetD = tetF - Pi / nCell
lF = rhoF * tetF;
lD = rhoD * tetD;
eF = tetF - betF
eD1 = tetF - betF
eD2 = - bet0
ndxF = -K rhoF / rF
ndxD = K rhoD / rD
(* l0 = rF  Cos[betF] - rhoF  Sin[tetF] *)
l0 = rD Sin[bet0]
```

```
0.156333

6.47319

6.1582

0.120222

0.141741

0.141741

-0.0108331

-15.6586

14.9121

0.748249
```

```
psi := 0;
wedge[rho_, epsi_] =
  {{1, 0, 0, 0, 0, 0}, {Tan[epsi] / rho, 1, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0},
   {0, 0, -Tan[epsi - psi] / rho, 1, 0, 0}, {0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 1}};
(*wedge[rhoF,eF];
 %//MatrixForm wedge[-rhoD,-eD1];
 %//MatrixForm wedge[-rhoD,-eD2];
 %//MatrixForm*)
bend[teta_, rho_, ndx_] = {{Cos[teta rho Sqrt[(1 - ndx) / rho / rho]],
    Sin[teta rho Sqrt[(1 - ndx) / rho / rho]] / Sqrt[(1 - ndx) / rho / rho], 0, 0,
    0, (1 - Cos[teta rho Sqrt[(1 - ndx) / rho / rho]]) / (1 - ndx) / rho / rho / rho},
   {-Sqrt[(1 - ndx) / rho / rho] Sin[teta rho Sqrt[(1 - ndx) / rho / rho]],
    Cos[teta rho Sqrt[(1 - ndx) / rho / rho]], 0, 0, 0,
    Sin[teta rho Sqrt[(1 - ndx) / rho / rho]] / Sqrt[(1 - ndx) / rho / rho] / rho},
   {0, 0, Cos[teta rho Sqrt[(ndx) / rho / rho]],
    Sin[teta rho Sqrt[(ndx) / rho / rho]] / Sqrt[(ndx) / rho / rho], 0, 0},
   {0, 0, -Sqrt[(ndx) / rho / rho] Sin[teta rho Sqrt[(ndx) / rho / rho]],
    Cos[teta rho Sqrt[(ndx) / rho / rho]], 0, 0}, {0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 1}};
(*bend[tetF,rhoF,ndxF];
 %//MatrixForm bend[-tetD,-rhoD,ndxD];
 %//MatrixForm*)
drift[L_] = {{1, L, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0},
   {0, 0, 1, L, 0, 0}, {0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 1}};
(*drift[l];
 %//MatrixForm*)

s0 = drift[10];
eD2;
rhoD;
wD2 = wedge[-rhoD, -eD2];
eD1;
rhoD;
wD1 = wedge[-rhoD, -eD1];
eF;
rhoF;
wF = wedge[rhoF, eF];
ndxD;
BD = bend[-tetD, -rhoD, ndxD];
ndxF;
BF = bend[tetF, rhoF, ndxF];
prod = FullSimplify[s0.wD2.BD.wD1.wF.BF.BF.wF.wD1.BD.wD2.s0] ;
% // MatrixForm
prod[[3]][[4]];
(prod[[1]][[1]] + prod[[2]][[2]]) / 2;
phaseXcell = ArcCos[0.5 * (prod[[1]][[1]] + prod[[2]][[2]])] / (2 Pi)
phaseXcell * 360
totalQx = phaseXcell * nCell
phaseZcell = ArcCos[0.5 * (prod[[3]][[3]] + prod[[4]][[4]])] / (2 Pi)
phaseZcell * 360
totalQz = phaseZcell * nCell
```

```
data = Import["betaFunctions.eps"]
Show[Graphics[data]]
```



ffag betaBeam

To summarize, typical parameters of a radial sector FFAG :

energy range	(MeV/u)	40 → 500
rigidities	(T.m)	2.8 → 10.9
geometrical radius	(m)	80
number of cells		60
r/z cell phase adv., typical		$2\pi/3$ / $\pi/3$
K		200
B max.	(T)	1.8
cell length	(m)	8
drift length	(m)	4
orbit excursion	(m)	0.5

quite large ring...

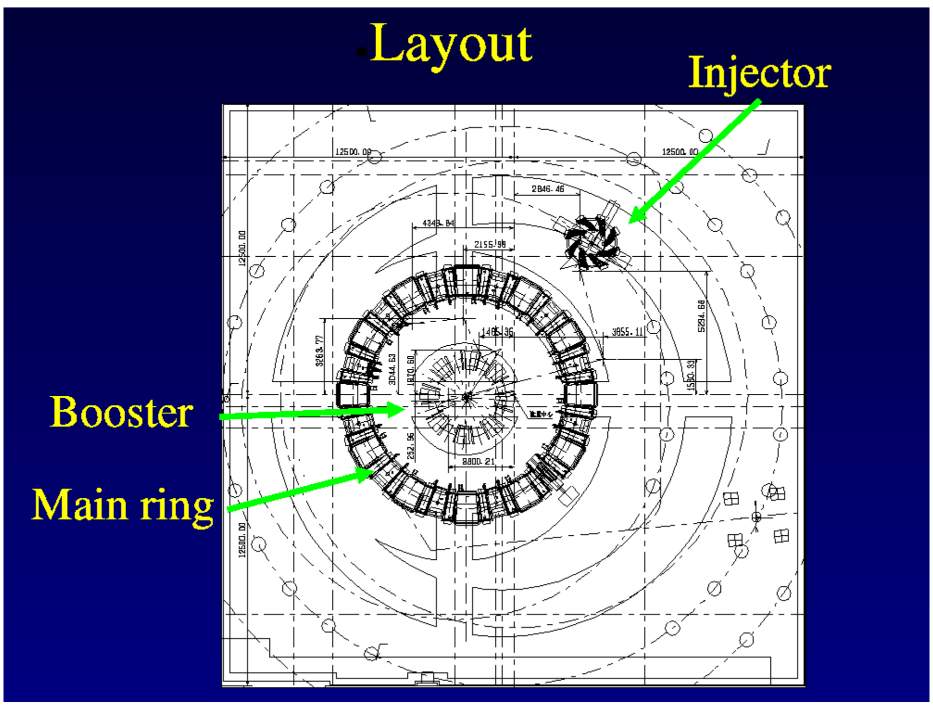
SPIRAL SECTOR FFAG

Yields more compact ring due
to absence of negative dipole.

Note :
design of a fourth stage for KURRI facility,
1 GeV, already exists

Typical parameters of a spiral sector FFAG – equivalent C 400 MeV/u :

energy range, ^{6}He	(MeV/u)	30 → 200
rigidities	(T.m)	2.3 → 6.4
circumference	(m)	45
geometrical radius	(m)	7.2
number of cells		16
K		12
B max.	(T)	2
orbit excursion	(m)	0.5



THE OTHER WAY : NON−SCALING FFAG

Yields more compact magnets due to small D_x, and compact machine
(comparable to regular snchrotron).

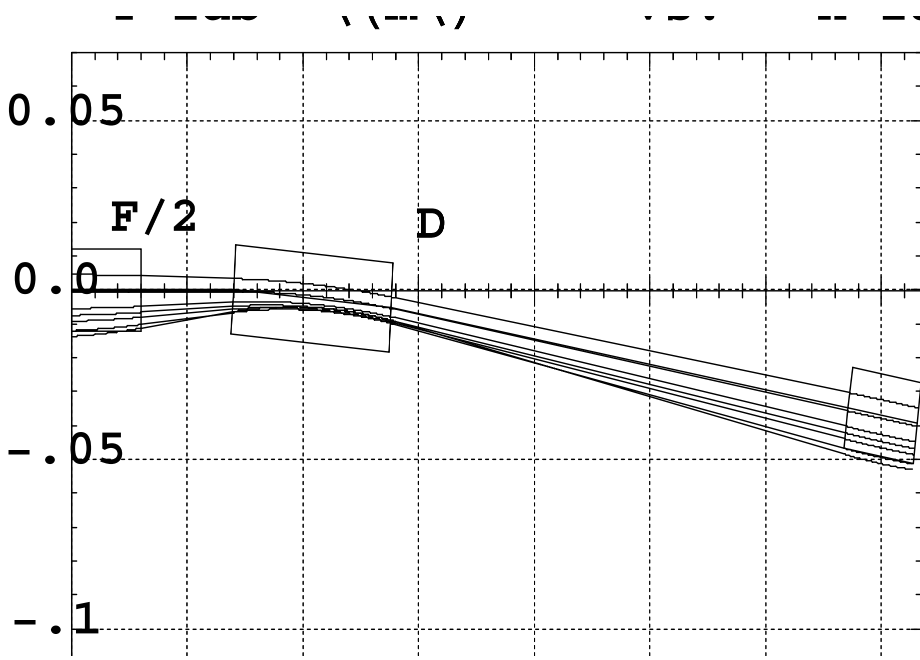
Already studied for muons (NuFact)

Two types of lattices could be envisaged :

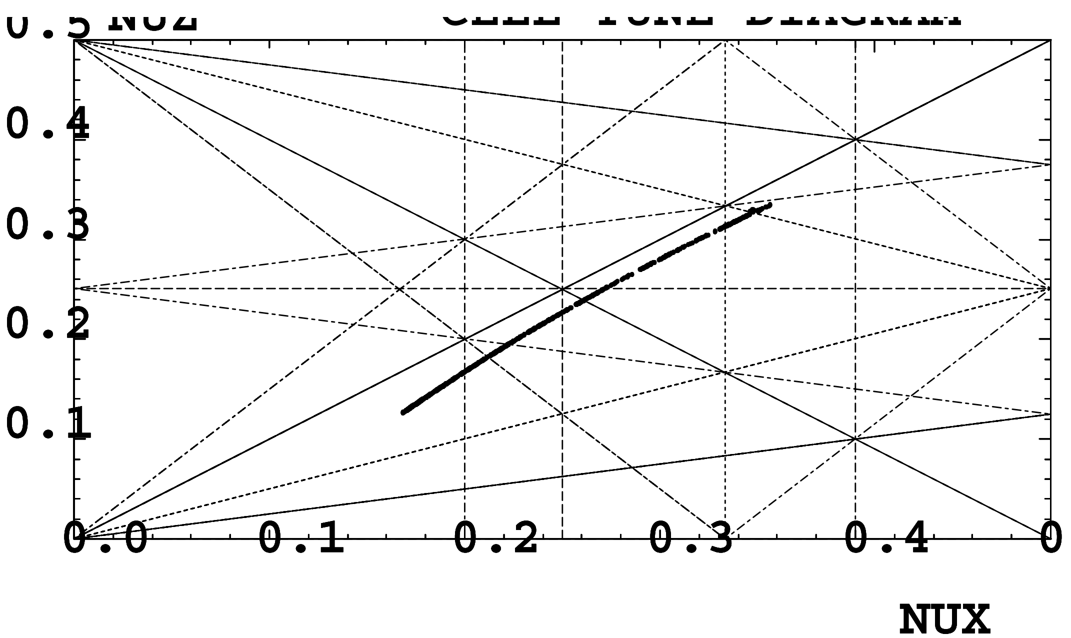
Linear lattice

Tune vary in the course of acceleration. Crossing of several integer and 1/2-integer resonances.

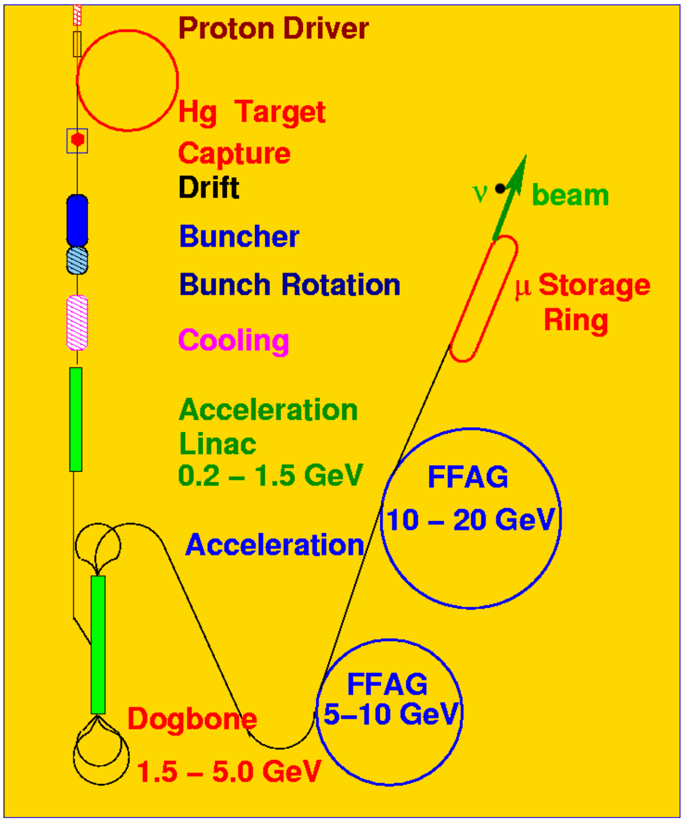
```
data = Import["closedOrbit.eps"]
Show[Graphics[data]]
```



```
data = Import["tuneDiagPath.eps"]
Show[Graphics[data]]
```



```
data = Import["feb17-study2a-schematics2.gif"]
Show[Graphics[data]]
```

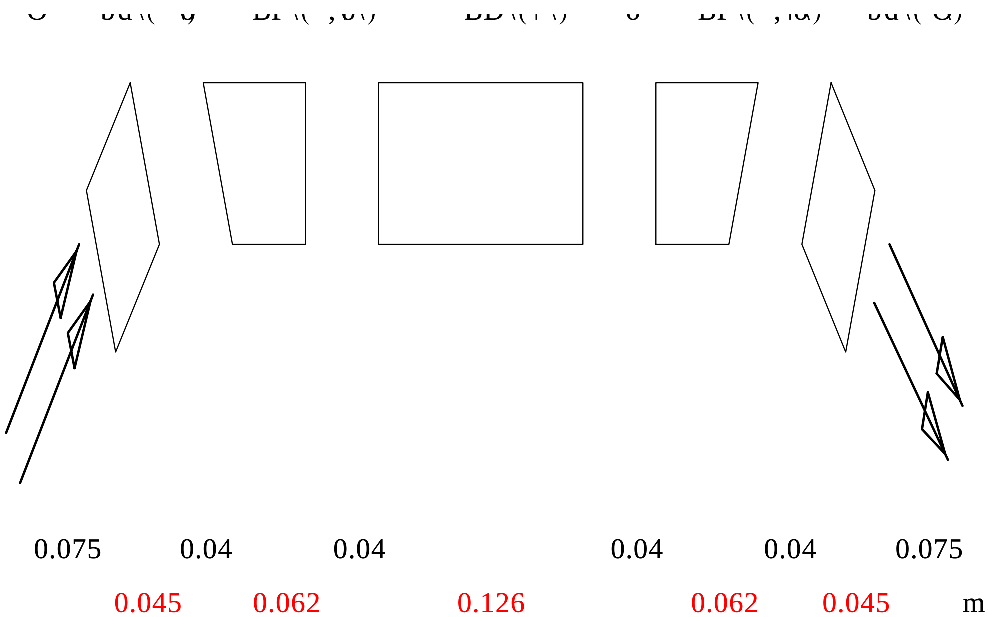


10 → 20 FFAG ring in US Study IIa :
10 turns acceleration
91 cells, 426 m circumference
field in F/D quadrupoles : 4.4 / 6.9 T

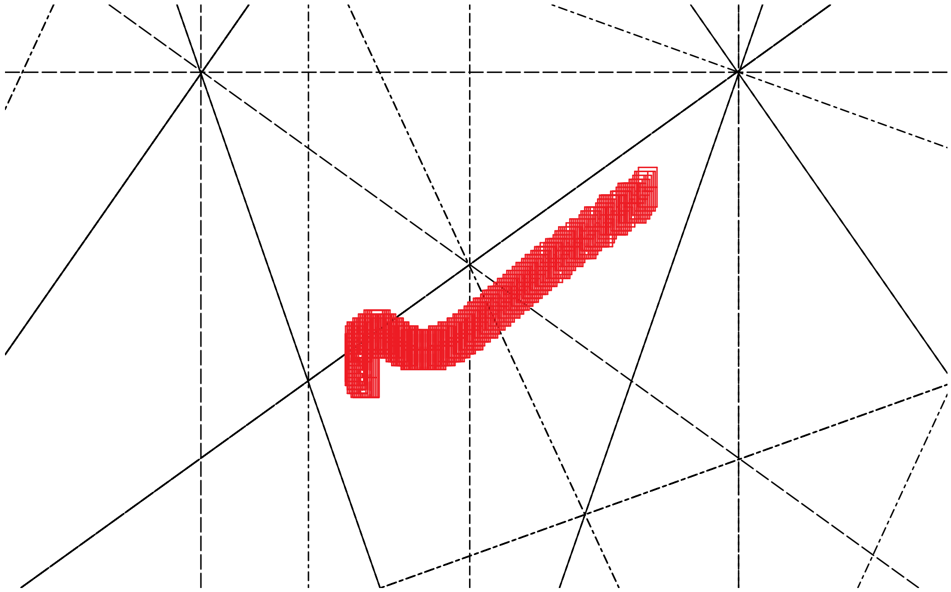
Non linear lattice
Better control on tunes : ν_x constant and ν_z change is small.

Possibility of insertion sections (for RF, inj/extraction, collimation)

```
data = Import["Fig1.eps"]
Show[Graphics[data]]
```



```
data = Import["Fig10.eps"]
Show[Graphics[data]]
```



In both cases, linear and non−linear lattices, issues :

Fast acceleration
Resonance crossing
Acceptance

A project of an electron model has been launched
A proton model (40 MeV) is in discussion